Replacement Workers and Collective Bargaining Outcomes in the Public Sector: The Case of School Teachers*

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Abstract

This paper studies the effect of replacement teachers on regular school teachers' wages, strike likelihood, and social welfare. Our findings suggest that policies that permit the use of replacement teachers in case of a strike increase the bargaining power of the school board. As a consequence, the probability of a strike and the settlement wages of regular teachers are reduced. We show that these policies have a positive welfare effect, and that the increase in replacement teachers' wages is also a welfare improving policy.

KEYWORDS: Collective Bargaining; Strikes; Replacement Workers; Non-Cooperative Games; Asymmetric Information

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1 Introduction

Substitute teachers are extensively employed by US school districts to ensure the provision of the number of educational days established by law in case of teacher absenteeism (due to sickness, professional development, etc.). They are also employed to replace regular teachers in case of strikes,[1] acting as "replacement teachers."[2] The use of replacement teachers by US school districts in case of regular teachers strikes have been amply documented in the media.[3] Replacement teachers may affect the balance of power between the school board and the regular teachers' union in collective bargaining negotiations. Then, they may affect regular teachers' settlement wages, strike likelihood and hence, social welfare. However, no formal study on the effect of replacement teachers on regular teachers' collective bargaining outcomes has been conducted. Our study attempts to fill this gap.

Our paper theoretically assesses the effect of replacement teachers on regular teachers' settlement wages, the likelihood of strikes, and social welfare. Our framework constitutes the first attempt to model the sequential wage negotiation in public schools by explicitly including the use of replacement teachers.

An interesting example of the influence of replacement on regular teachers' collective bargaining outcomes is provided by the change in the Pennsylvania collective bargaining statute of public school teachers in 1992 and its effects on wages and strikes. Before 1992, the use of selective strikes, i.e., sporadic short-term strikes, by the Pennsylvania teachers' union prevented school boards from hiring replacement teachers in case of a strike.[4] As a consequence, instructional days were lost and complains from the general public were widespread. In July 1992, the statute regulating the

[1]Nine US states allow teachers to strike (California, among others). School strikes are forbidden in 24 states. However, several of those states do use strikes as bargaining tools (see for example, the case of New York school districts). Finally, 17 states have no collective-bargaining laws.

[2]The term "replacement teachers" used here does not refer to practices of school districts related to hiring, in case of a strike, unqualified scab substitutes by waiving many hiring requirements for substitute teachers. It refers to the use of substitute teachers in case of a strike. In most US school districts, substitute teachers do not pertain to the regular teachers' union. Only in July 2000, the National Substitute Teachers Alliance, the first nationwide organization for substitute teachers, was formed. By January 2001, only 5 percent of substitutes had been represented by a bargaining unit (The School Administrator Web Edition, January 2001).

[3]See for example, Galbally (2002) for the case of Minnesota school districts; Oregon School Board Association Website (http://www.osba.org), for the case of Oregon school districts, among others.

[4]This selective-strikes strategy used by school teachers in Pennsylvania was characterized by the union notification to the school district each night whether teachers would report for work next day. "Striking teachers who decide to work on a day-to-day basis say a selective strike calls to their cause, protects their income and prevents school districts from hiring replacement" (Pittsburgh Post-Gazette, 1990).

collective bargaining negotiations of public school teachers was modified by prohibiting the use of selective strikes, and hence, allowing school boards to hire replacement teachers in case of a strike.[5] "[T]he passage of Act 88 of 1992 finally restored some balance to the bargaining process, dramatically reducing the number of strikes ... and controlling salary increases" (Pennsylvania School Board Association, 1997).[6]

Several studies have been conducted on the effect of replacement workers on strike likelihood and wages for private sector firms. Cramton, Gunderson and Tracy (1999) find that a policy that bans the use of replacement workers increases the likelihood of strikes in Canadian private firms. In addition, their results suggest that the prohibition of the use of replacement workers generates higher settlement wages. Cramton and Tracy (1998) study the effect of replacement workers on strike likelihood in U.S. private firms. Their empirical findings suggest that the policy that provides employers the right to hire replacement workers accounts for half of the decline in strike activity during the 1980s. They also construct a bargaining model that incorporates the effect of replacement workers on dispute likelihood. Their model assumes linearity in the objective function of the employer, i.e., that the employer's monetary valuation of the product of labor is proportional to the utilized amount of labor (i.e., constant marginal product of labor).[7] Note that in case of the provision of education, the school board's monetary valuation of the loss generated by one additional day of strike (marginal loss) may be non-constant and may depend on the number of previous strike days. Then, a setup that does not impose linearity in the objective function of the employer might be more suitable for the study of public school wage bargaining. Previous studies of collective bargaining in the public sector have been focused on employment and wage determination,

[5]In addition, mandatory non-binding arbitration was adopted. Empirical evidence on labor negotiations and pre-trial bargaining suggests that non-binding arbitration does not diminish the dispute rate because non-binding arbitration does not change the parties' incentives to reveal information. Hence, if any effect on collective bargaining outcomes can be observed as a product of this reform, it should be expected to come from the prohibition of selective strikes, which allowed school boards to hire replacement teachers in case of a strike.

[6]"In 1991-92 (the last full year under [the previous statute]), there were 36 teacher strikes. In 1992-93 (the first year under Act 88), that number dropped by more than half, to 17. In 1995-1996, there were only five teacher strikes in Pennsylvania. Salary increases negotiated under Act 88 have drastically declined each year. For example, third year increases in contracts negotiated in 1991 (the last full year before Act 88 took effect) averaged 7.09 percent; those negotiated in 1993 (the first year under the new law) averaged 5.17 percent; and those negotiated in 1996 averaged just 3.76 percent ... Act 88 clearly is enabling school boards to bargain contracts that are more in line both with private sector salaries for comparable positions as well as with the ability of their communities to pay ... Clearly, Act 88 has worked to level the bargaining table in public school negotiations, producing more economical settlements more quickly than ever before and more in line with community norms" (Pennsylvania School Board Association, 1997).

[7]Cramton and Tracy (1994) analyze time-varying strike costs in private sector. However, this setup is not used to assess the effect of substitute workers.

using theoretical frameworks that do not allow for sequential bargaining or incomplete information (see for example, Currie, 1991; Tracy and Gyourko, 1991). To the best of our knowledge, however, no sequential wage bargaining model has been developed to explain wage negotiations in the public sector.

The model developed in this paper belongs to the family of the screening models of sequential wage bargaining. We first present a sequential game theoretic model of incomplete information, which generalizes the objective function of the school board (the employer), making it suitable to study collective bargaining in the public sector. Specifically, the assumption of linearity of the objective function of the employer, used in models of private sector wage bargaining, is relaxed. In common with other screening models, our model predicts the negative relationship between the strike length and the settlement wages without the unrealistic assumption of full commitment of the union to a resistance curve, used in static models such as Card (1990).[8] In addition, our model assumes sequential bargaining between the school board and the teachers' union. This bargaining consists of series of offers by the union, which can be accepted or rejected by the school board. There is a fixed time interval between the offers. Rejection of the union's offer implies that there will be a strike for at least one period, i.e., until the next offer. As long as the strike continues, every period the union will update its beliefs about the school board's type (i.e., capacity to pay) and will lower its wage demand.[9] Second, we explicitly model the use of replacement teachers in case

[8] The "union's commitment to a resistance curve" framework implies that the union offers the firm a "menu" of combinations of strike length and wages. If the profitability of the employer is high, it accepts the no-strike-high-wage combination. If the employer expects low profits, it chooses the long-strike-low-wage option. This approach has two advantages. First, it allows a modeler to abstract from the mechanical features of the bargaining process, such as the sequential order of offers and counteroffers, and the time between offers. Second, this approach allows for a tractable modeling of risk-aversion of the union (used in Card, 1990), while all other approaches have to assume risk-neutrality for the sake of mathematical tractability. However, the assumption of full commitment to a resistance schedule is questionable, because such a commitment is not an equilibrium behavior for the union. Once the employer chooses the long-strike-low-wage option, the union would be better off by returning to work before the committed days of strike are accomplished. In other words, the union has an incentive to deviate.

On the other hand, sequential bargaining models, as our model, "spread out the union commitment in the sense that the union only commits itself not to renegotiate until the beginning of the next bargaining round" (Tracy, 1987; p. 153).

[9] The results derived from screening models, including our model, are sensitive to the time length between consecutive offers, known in the literature as the Coase conjecture. Gul, Sonnenschein and Wilson (1985) show that, as the length of time between bargaining rounds becomes arbitrarily small, the probability of delay goes to zero. That is, by allowing the union to bargain continuously, they demonstrate that one-sided asymmetric information models alone are not sufficient to generate bargaining delays (i.e., strikes). As stated by Tracy (1987), a direction to address Gul, et al. (1985) critique is to investigate whether commitment over some degree of time lapse between offers is reasonable. Hart (1986) suggests that delays may be the result of transaction costs in making offers. "[E]valuation of offers may involve different individuals, both for the union and for the firm. The need to confer may necessitate delays in order to arrange the appropriate meetings. [...] As a consequence, even though bargaining may take place continuously, meaningful offers will be made only at [discrete times]. In this case, there is no loss of generality in

of a strike. Our findings suggest that policies that permit the use of replacement teachers in case of a strike increase the bargaining power of the school board. As a consequence, the probability of a strike and the settlement wages of regular teachers are reduced.

Implications from this study are related to the effects of policies that permit the use of replacement teachers in case of a strike,[10] and to the effects of improving replacement teachers' wages.[11] It might be natural to think that policies that permit the use of replacement teachers and the improvement of replacement teachers' wages should have opposite effects because policies that permit the use of replacement teachers raise the bargaining power of the school board, while increases in replacement teachers' wages lower the bargaining power of the board (because they make it more costly to hire replacement teachers). We show, however, that, both policies reduce the probability of a strike and increase the social welfare.

Specifically, policies which facilitate the use of replacement teachers in case of a strike make strikes less costly for the school board, and hence reduce the likelihood that the school board accepts the initial wage demand of the union and increases the probability of a strike (direct effect). Anticipating the behavior of the school board, unions reduce their wage demand, which reduces the probability of a strike (indirect effect). We show that the indirect effect dominates. Hence, these policies lower the probability of a strike and increase the social welfare. On the other hand, an increase in the wages received by replacement teachers makes strikes more costly for the school board and hence, increases the likelihood that the school board accepts the initial wage demand of the union and reduces the probability of a strike (direct effect). Anticipating the reaction of the school board, unions increase their wage demands, which will increase the probability of a strike (indirect effect). We show, however, that the direct effect dominates. Hence, an increase in the wages received by replacement teachers lowers the probability of a strike and increases the social

working with a dynamic model with discrete rounds" (Tracy, 1987; p. 154).

[10]Examples of these policies are those that relax the requirement of working in the school district for a certain period of time prior to the strike to qualify as a replacement teacher. This is a requirement currently in place in several states. Strong requirements on academic credentials and experience are necessary for assuring quality in the provision of educational services. Teaching experience, however, can be obtained in another school district without jeopardizing the quality of education.

Policies that prohibit the use of selective strikes and those that relax a total ban on the use of replacement teachers are additional examples.

[11]Currently, replacement teachers wages vary widely nationwide, from less than \$40 per day in a few rural areas to \$150 in cities such as Los Angeles. "And subs in most places say they need higher pay, better training and better benefits if they're going to continue picking up the phone" (Toppo, 2001). "As for pay, it averages about \$65 to \$70 a day nationally. "In some parts of the country, it's as low as \$45 a day - about what a salon stylist would make for taking a few inches off the bottom of a customer's hair" (The Christian Science Monitor, 2002).

welfare.

The rest of the paper is organized as follows. Section Two outlines the basic theoretical model of wage bargaining in public schools. Section Three extends the basic model by incorporating the replacement teachers. Section Four presents a welfare analysis for the model with replacement teachers. Section Five discusses the main policy implications of this study. Section Six concludes the paper and suggests directions for future research.

2 A Wage Bargaining Model for the Public Sector: The Case of School Teachers

2.1 Model Setup

Consider a bargaining game about wages between an informed party, the school board (i.e., the employer),[12] and a uniformed party, the regular teachers' union. The source of asymmetry is the future capacity to pay of the school board, θ. Future capacity to pay is defined as the total funds that the school district can allocate to pay teacher wages during the next contract period. The value of θ is private information for the school board and represents its type. We assume that school boards have better information about the availability of future funds to be allocated to pay teacher wages: 1) feasibility of obtaining additional local funds (by increasing the current property tax rate); and, 2) possibility of reallocating current funds from non-wage expenditures to teacher wages. The union does not know the realization of θ, but it knows that it is distributed uniformly over the interval $[\theta_L, \theta_H]$.

Assume the following objective function for the school board

$$U = v(\theta, L) - wL, \tag{1}$$

where $v(\theta, L)$ represents the school district's monetary valuation of the provision of L periods of instruction and wL is the monetary cost of the labor needed to provide L periods of instruction. The maximum number of periods of instruction is determined by law. Then, the union and the school board bargain only over the wages. The maximum number of periods of instruction is not equal to the effective number of periods of instruction if a strike occurs. We abstract in the model from make-up periods, where teachers can recover the periods lost due to strikes. The quality of education is what matters for the community and therefore, for the school district. Even if strike

[12] We use the terms "school board" and "school district" interchangeably.

periods can be recovered, the quality of education is already compromised by the interruption in the provision of educational services due to the strike. We assume that each period of instruction requires the same amount of labor. Therefore, it is possible to normalize the amount of labor to 1 and consider L also as the total amount of labor used in the provision of L periods of instruction.

The rationale for the school board's maximization of the difference between the valuation of labor services and the cost of these services relies on the budget constraint that a school district confronts. As previous empirical studies on public schools find, "[w]hile public sector organizations have no profit motive, school boards operate under cost constraints based on the limits of federal and state funding/revenue sharing, as well as the ability and willingness of the local community to fund education through increased taxes" (Zwerling and Thomason, 1995 p. 469; see also Dilts, 1986).

For the sake of mathematic tractability, we consider that the school board's monetary valuation of the provision of education is linear in θ.

$$v(\theta) = \theta u(L) \tag{2}$$

and therefore,

$$U = \theta u(L) - wL. \tag{3}$$

We assume that $v(\theta) > 0$, i.e., the monetary valuation of provision of education is positively related to the school board's type. The rationale behind this positive relationship is the following: if the school district's capacity to pay increases (due to an increase in the community willingness to support education via taxes), it is expected that the school board would respond to its community demands by increasing its monetary valuation of the provision of education. Thus, a larger θ will imply higher losses due to a strike and greater willingness to accept higher settlement wages in order to avoid strikes. We also assume that $v_L > 0$, i.e., the school district's valuation of periods of instruction is strictly increasing in the number of periods of instruction.

We keep the general form of $u(L)$, allowing for a non-constant marginal valuation of labor,[13] only assuming that $u(L)$ is continuously differentiable and common knowledge. We assume that

[13]The school board's objective function allows for a non-constant loss due to strike, i.e., given that $u(L)$ can be linear, convex or concave, the valuation function $\theta u(L)$ can be also linear (constant marginal monetary benefit from provision of education–or constant marginal monetary loss from strike), convex (increasing marginal monetary benefit from provision of education–or decreasing marginal monetary loss from strike), or concave (decreasing marginal monetary benefit from provision of education–or increasing marginal monetary loss).

the union is risk-neutral and that the union maximizes the expected value of the wage bill. Union members have a reservation wage $\bar{w}$, which represents their income in the case of a strike. We do not restrict, however, the risk behavior of the school district.

We also assume that the maximum number of periods of instruction is $N+2$, $N \geq 1$.[14] To keep algebra manageable, we have also assumed that the bargaining lasts for 3 periods and that the union can make an offer to the employer only twice: at the beginning of the first and the second periods. If both offers are rejected, the employer makes a single counteroffer at the beginning of the third period. Thus the model is essentially a screening game. The credible threat of a strike allows the union to wage discriminate, i.e., to force school boards more averse to strikes to accept higher wages. After two strike periods, given (4) and the union's income in case of strike, the school board proposes the reservation wage $\bar{w}$ and the union accepts the proposal. The counteroffer by the board ensures that in equilibrium the schools are never closed for more than 2 periods. This feature of the model has an empirical rationale: it rules out long strikes, which rarely take place.

Assume also

$$\theta_L[u(N+1+i) - u(N+i)] > \bar{w}, \tag{4}$$

for $i = 0, 1$. The school board's valuation of one additional period (day, week, etc.) of instruction is higher than the reservation wage. In other words, a school board never closes the school even for one period, if the union is willing to accept the reservation wage.

It is important to note that the functional form of $v(\theta)$ allows for the objective function of a for-profit firm as a special case: If $\theta u(L)$ represents the total revenues of a firm, $[\theta u(L) - wL]$ would be the profits. We do not make any restrictive assumptions about the function $u(L)$, except that it is strictly increasing and continuously differentiable. Hence this setup is also useful for analysis of the collective bargaining negotiations in private sector, for those firms for which the standard assumption of linear valuation of labor is not valid.

2.2 Model Solution

The sequential structure of the game allows the regular teachers' union to update its beliefs about the board's type depending on its response to the union's offers. The larger θ, the lower the length of strike the board is willing to endure to benefit from lower wages.

[14] Our specification of L allows to measure periods of instruction in different units: days, weeks, months, etc.

The game proceeds as follows. The first offer of the union, w_2, is the highest one. The board accepts the offer if its θ is relatively high, i.e., if θ pertains to the interval $\theta_2 \leq \theta \leq \theta_H$. The school board of type θ_2 is indifferent between accepting the offer or incurring in one-period strike, i.e., its expected payoffs are the same in both situations. If the first offer is rejected, the union updated its beliefs. Now, it believes that θ is distributed uniformly over the interval $[\theta_L, \theta_2]$ and reacts with a lower second offer, w_1. This offer is accepted by the board, if its θ belongs to the intermediate range of types $[\theta_1, \theta_2]$. The board of the type θ_1 is indifferent between accepting w_1 and the strike continuation, i.e., its expected payoffs are the same in both situations. If the second offer is rejected, the union updates its beliefs. Now it believes that θ is distributed uniformly over the interval $[\theta_L, \theta_1]$. Then, the school board counteroffers $\bar{w}$ and the union accepts. Only the board types that pertain to this interval are prepared to endure a two-period strike in order to benefit from the lowest settlement wage, $\bar{w}$.

Two-period strikes are possible only if there is sufficient uncertainty about the type of the school board, i.e., if $\theta_L < \theta_1$. This section considers this case.[15]

The solution concept adopted here is the Perfect-Bayesian equilibrium and the model is solved using backward induction.

The union decides its second offer, w_1, to maximize its expected payoff

$$\Pi_2 = \frac{\theta_2 - \theta_1}{\theta_2 - \theta_L}(N+1)w_1 + \frac{\theta_1 - \theta_L}{\theta_2 - \theta_L}(N+1)\bar{w}. \tag{5}$$

The school board of type θ_1 is indifferent between accepting and rejecting the offer if its expected payoffs are the same in both situations. Equation (6) describes the indifference condition.

$$\theta_1 u(N+1) - (N+1)w_1 = \theta_1 u(N) - N\bar{w}. \tag{6}$$

The union chooses w_1 in order to maximize (5) subject to (6). Maximization yields

$$w_1 = \frac{1}{N+1}\left[\frac{\theta_2[u(N+1) - u(N)]}{2} + (N+0.5)\bar{w}\right], \tag{7}$$

and replacing w_1 in (5) we get

$$\theta_1 = \frac{\theta_2}{2} + \frac{\bar{w}}{2[u(N+1) - u(N)]}. \tag{8}$$

[15]For the solution of the game with lower uncertainty, see Appendix B.

Lemmas 1 and 2 in Appendix verify that $\theta_1 < \theta_2$ and $w_1 > \bar{w}$.

Now consider the optimal strategies in period 1. The school board of type θ_2 will be indifferent between accepting the offer w_2 without a strike, and enduring a one-period strike and then accept the offer w_1, only if its expected payoffs are the same in both situations. The indifference condition implies that

$$\theta_2 u(N+2) - (N+2)w_2 = \theta_2 u(N+1) - (N+1)w_1. \tag{9}$$

The union chooses w_2 to maximize its expected payoff, represented by equation (10), subject to the board's indifference condition, expressed by equation (9).

$$\Pi_1 = \frac{\theta_H - \theta_2}{\theta_H - \theta_L}(N+2)w_2 + \frac{\theta_2 - \theta_L}{\theta_H - \theta_L}\left[\frac{\theta_2 - \theta_1}{\theta_2 - \theta_L}(N+1)w_1 + \frac{\theta_1 - \theta_L}{\theta_2 - \theta_L}(N+1)\bar{w} + \bar{w}\right], \tag{10}$$

where w_1 and θ_1 are represented by equations (7) and (8).
Optimization yields

$$w_2 = \frac{1}{N+2}[\theta_2(u(N+2) - 0.5u(N+1) - 0.5u(N)) + (N+0.5)\bar{w}]. \tag{11}$$

Replacing w_2 in the board's indifference condition and solving for θ_2, we get

$$\theta_2 = \frac{\theta_H[u(N+2) - 0.5u(N+1) - 0.5u(N)] + \bar{w}}{2[u(N+2) - 0.75u(N+1) - 0.25u(N)]}. \tag{12}$$

Lemmas 3 and 4 in Appendix verify that $\theta_2 < \theta_H$ and $w_2 > w_1$.

Finally, by substituting θ_2 into (8), we obtain a necessary and sufficient condition for the existence of the equilibrium with a two-period strike.

$$\theta_1 = \frac{\theta_H[u(N+2) - 0.5u(N+1) - 0.5u(N)] + \bar{w}}{4[u(N+2) - 0.75u(N+1) - 0.25u(N)]} + \frac{\bar{w}}{2[u(N+1) - u(N)]} > \theta_L. \tag{13}$$

The following propositions state the implications of the basic model. Proposition 1 indicates the relationship between wages and the strike length. Proposition 2 presents the effect of the union's outside opportunities on its settlement wage proposal.

Proposition 1: There is a negative relationship between the settlement wages and the strike length.

Proof. See Appendix A.

Proposition 2: There is a positive relationship between the union's settlement proposal and the reservation wages, $\bar{w}$.

Proof. See Appendix A.

Proposition 3 shows the effect of the outside opportunities of the union on the probability of a strike.

Proposition 3: There is a positive relationship between the reservation wages, $\bar{w}$, and the probability of a strike.

Proof. See Appendix A.

Propositions 4 and 5 summarize the effect of an increase in the school board's capacity to pay on expected wages and strike likelihood. We model such an increase as a parallel upward shift of the distribution of θ. Mathematically, we assume that the school board "type" is distributed uniformly over the interval $[\theta_L + \Delta, \theta_H + \Delta]$, and analyze the impact of a change in Δ on the strike probability and expected wages. The parallel shift of the whole distribution ensures that the results are not caused by the change in uncertainty about the board's type.

Proposition 4: An increase in the school board's capacity to pay reduces the probability of a strike.

Proof. See Appendix A.

Proposition 5: An increase in the school board's capacity to pay increases the expected settlement wages.

Proof. See Appendix A.

An additional prediction is that the increase in uncertainty generates higher strike length. This result comes from the fact that a two-period strike is possible only if there is sufficient uncertainty about the type of the school board, i.e., $\theta_L < \theta_1$. Under a very low uncertainty, no strike is possible (see Appendix B for details about the solution of the model under low uncertainty).

3 Replacement Teachers: An Extension of the Benchmark Model

We will now consider the effect of replacement teachers on regular teachers' wages and strike likelihood. Following empirical regularities, we assume that replacement teachers are less productive than regular teachers. In particular we assume that one hour (day, week, etc.) of services of a replacement teacher is the perfect substitute for α hours (days, weeks) of a regular unionized teacher, $\alpha \leq 1$. We also assume that replacement teachers earn $\gamma\bar{w}$ and that $\alpha < \gamma$. If this last assumption does not hold, the school board would benefit from a permanent substitution of the unionized teachers with the replacement teachers.

For the sake of mathematical tractability and without loss of generality, we assume that the bargaining lasts for 2 periods and that the maximum number of periods of instruction is $N + 1$, $N \geq 1$.[16] The game proceeds as follows. At the beginning of the first period, the union makes an offer. If the offer is rejected, the union calls a one-period strike. At the beginning of the second period the school board makes a counteroffer, $\bar{w}$, which is accepted with certainty. Figure 1 represents the sequence of moves.

[**INSERT FIGURE 1**]

If the school board endures a one-period strike, its utility is

$$U = \theta u(N + \alpha\beta) - \beta\gamma\bar{w} - N\bar{w}, \tag{14}$$

where β is the share of regular workers substituted for replacement teachers. We assume that $0 \leq \beta \leq \bar{\beta} \leq 1$. Here, $\bar{\beta}$ is the upper limit on the replacement share. We assume that this upper limit is determined by law or school board's policies.[17] In states where the collective bargaining statute facilitates the programming of replacement teachers by, for example, prohibiting selective strikes,[18] the upper limit on the replacement share will be higher. In case school boards impose too restrictive requirements for hiring replacement teacher, this upper limit will be smaller. In the extreme case of a ban on replacement teachers, the replacement share would be $\beta = \bar{\beta} = 0$.

We make two additional assumptions regarding the objective function of the school board and parameters of the model.

[16]The results from the benchmark model still hold under this model setup.

[17]We are focusing here on policies that permit the use of replacement teachers in case of a strike. Then, without loss of generality, we are abstracting from the effect that availability of replacement teachers, and therefore, γ, might have on $\bar{\beta}$. The qualitative results presented here, however, hold in case $\bar{\beta}$ depends on γ.

[18]See for example, the Pennsylvania's collective bargaining law, Act 88 of 1992.

$$\theta_L[u(N+1) - u(N+1-X)] > X\bar{w}, \tag{15}$$

for any X, such that $X \in (0, N+1)$. Intuitively, we assume that for any type of the school board, the welfare loss due the undersupply of education services exceeds the reservation wage of the unionized teachers. If this assumption were violated, some school boards might want to shorten the school year and/or fire some of teachers, even if they agree to receive just their reservation wage. Such an outcome never happens in reality, and hence this assumption is not restrictive.

Second, we assume that

$$\bar{w}^r > \alpha\bar{w}, \tag{16}$$

where $\bar{w}^r$ is the reservation wages of replacement teachers. In words, the reservation wages of the replacement teachers are no smaller than the reservation wages of the regular ones, or, if they are smaller, the difference is smaller than the difference in productivity.

The solution procedure and the equilibrium concept are the same as in the model without replacement teachers. The union chooses a wage offer, $\tilde{w}_1$ in order to maximize its expected payoff

$$\Pi_1 = \frac{\theta_H - \tilde{\theta}_1}{\theta_H - \theta_L}(N+1)\tilde{w}_1 + \frac{\tilde{\theta}_1 - \theta_L}{\theta_H - \theta_L}(N+1)\bar{w}, \tag{17}$$

subject to the constraint that the school board of type $\tilde{\theta}_1$ is indifferent between accepting the wage offer $\tilde{w}_1$ and enduring a one-period strike.

$$\tilde{\theta}_1 u(N+1) - \tilde{w}_1(N+1) = \tilde{\theta}_1 u(N+\alpha\beta) - \beta\gamma\bar{w} - N\bar{w}. \tag{18}$$

Optimization yields

$$\tilde{w}_1 = \frac{1}{N+1}[\tilde{\theta}_1(u(N+1) - u(N+\alpha\beta)) + (N+\beta\gamma)\bar{w}]. \tag{19}$$

Replacing $\tilde{w}_1$ in the indifference condition of the school board, we get

$$\tilde{\theta}_1 = \frac{\theta_H}{2} + \frac{(1-\beta\gamma)\bar{w}}{2[u(N+1) - u(N+\alpha\beta)]}. \tag{20}$$

Lemmas 5 and 6 in Appendix verify that $\tilde{w}_1 > \bar{w}$ and $\tilde{\theta}_1 < \theta_H$.

One-period strikes are possible only if there is sufficient uncertainty about the type of the school board, i.e., $\theta_L < \tilde{\theta}_1$. Then, a necessary and sufficient condition for the screening equilibrium with strikes is that

$$\tilde{\theta}_1 = \frac{\theta_H}{2} + \frac{(1-\beta\gamma)\bar{w}}{2[u(N+1) - u(N+\alpha\beta)]} > \theta_L. \tag{21}$$

If equation (21) does not hold, there is a pooling equilibrium where the union makes an offer acceptable to all school board types (see Appendix B for details about the solution of the model under low uncertainty).

Propositions 6 and 7 show the effects of replacement teachers on the strike likelihood and the regular teachers union's wage demand. We assume that the restriction $\beta \leq \bar{\beta}$ is binding. That is, we assume that $\beta = \bar{\beta}$, i.e., the replacement teachers share is constrained by legal or policy restrictions. If this restriction were not binding, β would not be affected by a marginal change in the upper limit on the replacement share $\bar{\beta}$.

More specifically, we assume that the effect of an increase in the replacement share β on the school board's expected payoff (from rejecting the union's offer and enduring a one-period strike) is strictly positive, i.e., the school board will always be willing to hire an additional replacement worker.

$$\frac{\partial}{\partial\beta}(\theta_L u(N+\alpha\beta) - \beta\gamma\bar{w}) = \theta_L \alpha u'(N+\alpha\beta) - \gamma\bar{w} > 0, \tag{22}$$

for any β such that $0 \leq \beta \leq \bar{\beta}$.

Proposition 6. If the replacement share limit is binding ($\beta = \bar{\beta}$), there is a negative relationship between this limit and the union's settlement wage proposal.

Proof. See Appendix A.

Proposition 7. If $u'(L)$ is a non-decreasing function of L, there is a negative relationship between the replacement limit and the probability of a strike.[19]

Proof. See Appendix A.

[19] The requirement of non-concavity of $u'(L)$ is reasonable for the case of the public school sector. The intuition is as follows. School boards are committed to provide the 180 days of instruction that the law mandates. The first day of strike, therefore, produces the highest loss. Once the strike has started and the legal requirement has been already violated, the loss of one extra day decreases as the number of previous days of strike increases. Therefore, $u(L)$ is a convex function, i.e., $u''(L) > 0$.

It is important to note that Proposition 7 states the sufficient but not the necessary condition for the negative relationship between replacement ratio and the likelihood of strike. In other words, even if $u(L)$ is a concave function, an increase in the replacement ratio may reduce the probability of a strike.

The result of Proposition 7 can be explained as follows. Policies that permit the use of replacement teachers in case of a strike (i.e., increase in the limit $\bar{\beta}$), reduces the cost of strike for the school board. It does not affect directly the cost of strike for the union because the model assumes that the replacement teachers are less efficient than the union members and therefore, the latter cannot displace them permanently. In this context, one might expect the probability of a strike to rise. However, there is an indirect effect of the increase in $\bar{\beta}$, which offsets the direct one: the regular teachers' union takes into account the reduction in the cost of strike for the school board and lowers the wage demand, $\tilde{w}$ (Proposition 6). This implies that a wider range of employers will find it optimal to accept the wage demand and avoid the strike.[20]

Propositions 8 and 9 summarize the effects of an increase in the wages for replacement teachers $\gamma\bar{w}$ (through an increase in γ) on the probability of a strike and the union's settlement wage proposal.

Proposition 8. There is a negative relationship between the replacement-wage parameter, γ, and the probability of a strike.

Proof. See Appendix A.

Intuitively, an increase in the wages of replacement teachers makes strikes more costly for the school board, and hence more types of school boards accept the initial wage demand of the union.

Proposition 9. There is a negative relationship between the replacement-wage parameter γ and the union's settlement wage proposal.

Proof. See Appendix A.

Intuitively, an increase in the wages of replacement teachers makes their employment less attractive for the school board, increases the cost of strike for the school board and hence, reduces the bargaining power of the school board. Hence, the union demands a higher wage.

4 Welfare Analysis

This section outlines the effects on social welfare of policies that permit the use of replacement teachers in case of a strike. We define the social welfare function, W, as the sum of the welfare

[20] Empirical findings on the effect of banning replacement workers on increasing the strike likelihood, reported in Cramton, Gunderson and Tracy (1999), support Proposition 7.

function for the school board, W^b, for the union, W^u, and for the replacement teachers, W^r.

$$W = W^b + W^u + W^r \tag{23}$$

Here, W^b is defined as

$$\begin{cases} \theta u(N+1) - (N+1)w & \text{if there is no strike} \\ \theta u(N+\alpha\beta) - \beta\gamma\bar{w} - N\bar{w} & \text{if there is a strike.} \end{cases} \tag{24}$$

W^u is defined as

$$\begin{cases} (N+1)w & \text{if there is no strike} \\ (N+1)\bar{w} & \text{if there is a strike.} \end{cases} \tag{25}$$

And, W^r is defined as

$$\begin{cases} \beta\bar{w}^r & \text{if there is no strike} \\ \beta\gamma\bar{w} & \text{if there is a strike,} \end{cases} \tag{26}$$

where $\bar{w}^r$ is the reservation wages of the replacement teachers.

Then, the social welfare function given a particular school board type θ, $W(\theta)$, is represented by

$$\begin{cases} W_{ns} = \theta u(N+1) + \beta\bar{w}^r & \text{if there is no strike} \\ W_s = \theta u(N+\alpha\beta) + \bar{w} & \text{if there is a strike.} \end{cases} \tag{27}$$

Hence, the expected social welfare function $E(W(\theta))$, given a particular school board type θ, $E(W(\theta))$ is given by

$$E(W(\theta)) = (1 - p_s(\theta))[\theta u(N+1) + \beta\bar{w}^r] + p_s(\theta)[\theta u(N+\alpha\beta) + \bar{w}], \tag{28}$$

where $p_s(\theta)$ represents the probability of a strike for the school board type θ.

The social welfare across school boards types is defined as

$$W = \int_{\theta_L}^{\theta_H} E(W(\theta))d\theta \tag{29}$$

Propositions 10 and 11 outline the positive effects on social welfare of policies that permit the use of replacement teachers in case of a strike and increases in replacement teachers' wages.

Proposition 10. If $u'(L)$ is a non-decreasing function of L, there is a positive relationship between the replacement limit $\bar{\beta}$ and social welfare.

Proof. See Appendix A.

Intuitively, policies that permit the use of replacement teachers in case of a strike lower the cost of strikes for the school board. School boards then are less willing to accept high settlement proposals from the union. Anticipating this behavior, the union lowers its settlement proposal. As a consequence, the likelihood of strike is reduced and the social welfare is increased.

Proposition 11. There is a positive relationship between the replacement-wage parameter, γ, and social welfare.

Proof. See Appendix A.

Intuitively, an increase in wages of replacement teachers affects social welfare only through the probability of a strike (equations 28 and 29), as these wages are just transfers from the school board to the teachers. An increase in γ makes a strike less likely (Proposition 8), as more types of school boards accept the initial wage demand of the union. Therefore, fewer strikes imply a higher social welfare.[21]

5 Policy Implications

Implications of this study are related to the effects of policies that permit the use of replacement teachers in case of a strike, and to increases in replacement workers' wages. It might be natural to think that both policies should have opposite effects: policies that permit the use of replacement teachers in case of a strike (an increase in $\bar{\beta}$) raise the bargaining power of the school board and reduce the wage demand of the union (Proposition 6) and the probability of a strike, while an increase in the wages received by replacement teachers (an increase in γ) lowers the bargaining power of the board and raises the wage demand of the union (Proposition 8) and might be expected to increase the probability of a strike. We show in this paper, however, that both policies reduce the probability of a strike and increase the social welfare.

Specifically, policies that permit the use of replacement teachers in case of a strike (i.e., an increase in $\bar{\beta}$) make strikes less costly for the school board, and hence reduce the likelihood that

[21]Note that in case $\bar{\beta}$ would depend on availability of replacement teachers, and therefore, on γ, there will be an additional effect of an increase on γ: an increase on availability of replacement teachers, and therefore, an improvement in the quality of the educational services due to a lower interruption of the provision of educational services during a strike. This last effect strengthens the positive welfare effect of an increase in replacement teachers' wages.

the school board accepts the initial wage demand of the union and increase the probability of a strike (direct effect). Anticipating the behavior of the school board, unions reduce their wage demand, which reduces the probability of a strike (indirect effect). We show that the indirect effect dominates. Hence, these last policies lower the probability of a strike and increase the social welfare. Note also that there is an additional effect of policies that permit the use of replacement teachers in case of a strike: no interruption in the provision of educational services, and therefore, better quality of education is provided in case of a strike. This last effect strengthens the positive welfare effect. On the other hand, an increase in the wages received by replacement teachers (i.e., an increase in γ) makes strikes more costly for the school board and hence, increases the likelihood that the school board accepts the initial wage demand of the union and reduces the probability of a strike (direct effect). Anticipating the reaction of the school board, unions increase their wage demands, which will increase the probability of a strike (indirect effect). We show, however, that the direct effect dominates. Hence, an increase in the wages received by replacement teachers lowers the probability of a strike and increases the social welfare.

6 Conclusions

This paper studies the effect of replacement teachers on regular school teachers' wages, strike likelihood, and social welfare. We model the collective bargaining negotiations between the school board and the regular teachers' union as a sequential game theoretic model of incomplete information, and explicitly include into the model the effect of replacement teachers.

Our findings suggest that policies that permit the use of replacement teachers in case of a strike increase the bargaining power of the school board. As a consequence, the probability of a strike and the settlement wages of regular teachers are reduced. In addition, we show that these policies and an improvement in replacement teachers' wages are welfare improving.

Appendix A. Proofs of Lemmas and Propositions

Proofs of Lemmas 1–6 and Propositions 1–11 follow.

Proofs of Lemmas

Lemma 1. $\theta_1 < \theta_2$.

Proof.

$$\theta_2 - \theta_1 =$$

$$\theta_2 - \left[\frac{\theta_2}{2} + \frac{\bar{w}}{2[u(N+1) - u(N)]}\right] = 0.5\left[\theta_2 - \frac{\bar{w}}{u(N+1) - u(N)}\right] > 0. \qquad (A1)$$

The last result follows from inequality (4). Q.E.D.

Lemma 2. $w_1 > \bar{w}$.

Proof. From equation (6),

$$w_1 = \frac{1}{N+1}\{\theta_1[u(N+1) - u(N)] + N\bar{w}\}. \qquad (A2)$$

Therefore,

$$w_1 - \bar{w} > \frac{1}{N+1}[\bar{w} + N\bar{w}] - \bar{w} = 0. \qquad (A3)$$

The last inequality follows from (4). Q.E.D.

Lemma 3. $\theta_2 < \theta_H$.

Proof. From equation (12),

$$\theta_H - \theta_2 =$$

$$\frac{2\theta_H[u(N+2) - u(N+1)] + 0.5\theta_H[u(N+1) - u(N)]}{2[u(N+2) - u(N+1)] + 0.5[u(N+1) - u(N)]} -$$

$$\frac{\theta_H[u(N+2) - u(N)] - 0.5\theta_H[u(N+1) - u(N)] - \theta_H\bar{w}}{2[u(N+2) - u(N+1)] + 0.5[u(N+1) - u(N)]} =$$

$$\frac{\theta_H[u(n+2) - u(N+1)] - \bar{w}}{2[u(N+2) - u(N+1)] + 0.5[u(N+1) - u(N)]} >$$

$$\frac{\bar{w} - \bar{w}}{2[u(N+2) - u(N+1)] + 0.5[u(N+1) - u(N)]} = 0. \qquad (A4)$$

Q.E.D.

Lemma 4. $w_2 > w_1$.

Proof.

$$w_2 - w_1 = \left[\frac{\theta_2[u(N+2) - u(N+1)] + 0.5\theta_2[u(N+1) - u(N)]}{N+2} + \frac{N+0.5}{N+2}\bar{w}\right] -$$

$$\left[\frac{\theta_2[u(N+1) - u(N)]}{2(N+1)} + \frac{N+0.5}{N+1}\bar{w}\right] >$$

$$\frac{\bar{w}}{N+2} + \left[\frac{1}{N+2} - \frac{1}{N+1}\right][0.5\theta_2[u(N+1) - u(N)] + (N+0.5)\bar{w}] >$$

$$\frac{\bar{w}}{N+2} - \frac{\bar{w}}{2(N+1)(N+2)} - \frac{\bar{w}(N+0.5)}{(N+1)(N+2)} = 0. \qquad (A5)$$

Q.E.D.

Lemma 5. $\tilde{w}_1 > \bar{w}$.

Proof.

$$\tilde{w}_1 - \bar{w} =$$

$$\frac{1}{N+1}[0.5\theta_H(u(N+1) - u(N+\alpha\beta)) + 0.5(1-\beta\gamma)\bar{w} + (N+\beta\gamma)\bar{w}] - \bar{w} >$$

$$\frac{1}{N+1}[0.5\bar{w}(1-\beta\gamma) + 0.5(1-\beta\gamma)\bar{w} + (N+\beta\gamma)\bar{w}] - \bar{w} = 0. \qquad (A6)$$

The inequality follows from (15). Q.E.D.

Lemma 6. $\tilde{\theta}_1 < \theta_H$.

Proof.

$$\theta_H - \theta_1 = \theta_H - \left[\frac{\theta_H}{2} + \frac{(1-\beta\gamma)\bar{w}}{2[u(N+1) - u(N+\alpha\beta)]}\right] =$$

$$\frac{\theta_H[u(N+1) - u(N+\alpha\beta)] - (1-\beta\gamma)\bar{w}}{2[u(N+1) - u(N+\alpha\beta)]} > 0. \qquad (A7)$$

The last result follows from inequality (15) . Q.E.D.

Proofs of Propositions

Proposition 1. There is a negative relationship between the settlement wages and the strike length.

Proof. Lemmas 2 and 4 show that $w_2 > w_1 > \bar{w}$. A one-period strike results in the settlement wage level w_1, while a two-period strike results in the settlement wage level $\bar{w}$. In case of no strike, the settlement wage is w_2. Q.E.D.

Proposition 2. There is a positive relationship between the union's settlement proposal and the reservation wages, $\bar{w}$.

Proof. Differentiating (12) with respect to $\bar{w}$ yields:

$$\frac{\partial \theta_2}{\partial \bar{w}} = \frac{1}{2[u(N+2) - 0.75u(N+1) - 0.25u(N)]} > 0. \qquad (A8)$$

Differentiating (11) with respect to $\bar{w}$,

$$\frac{\partial w_2}{\partial \bar{w}} = \frac{1}{N+2}[u(N+2) - 0.5u(N+1) - 0.5u(N)]\frac{\partial \theta_2}{\partial \bar{w}} + \frac{(N+0.5)}{N+2} > 0. \qquad (A9)$$

Finally, differentiating (7) with respect to $\bar{w}$,

$$\frac{\partial w_1}{\partial \bar{w}} = \frac{[u(N+1) - u(N)]}{2(N+1)}\frac{\partial \theta_2}{\partial \bar{w}} + \frac{N+0.5}{N+1} > 0. \qquad (A10)$$

Q.E.D.

Proposition 3. There is a positive relationship between the probability of a strike and the reservation wages, $\bar{w}$.

Proof. The probability of a strike, $p_s = \frac{\theta_2 - \theta_L}{\theta_H - \theta_L}$. Hence,

$$\frac{\partial p_s}{\partial \bar{w}} = \frac{1}{\theta_H - \theta_L}\frac{\partial \theta_2}{\partial \bar{w}} > 0. \qquad (A11)$$

Q.E.D.

Proposition 4. An increase in the employer's capacity to pay reduces the probability of a strike.

Proof.

Under a parallel shift of the distribution of θ,

$$\hat{\theta}_2 = \frac{(\theta_H + \Delta)[u(N+2) - 0.5u(N+1) - 0.5u(N)] + \bar{w}}{2[u(N+2) - 0.75u(N+1) - 0.25u(N)]} \qquad (A12)$$

and

$$\hat{p}_s = \frac{\hat{\theta}_2 - (\theta_L + \Delta)}{\theta_H + \Delta - (\theta_L + \Delta)} = \frac{\hat{\theta}_2 - (\theta_L + \Delta)}{\theta_H - \theta_L}. \qquad (A13)$$

Therefore,

$$\frac{\partial \hat{p}_s}{\partial \Delta} = \frac{\frac{\partial \hat{\theta}_2}{\partial \Delta} - 1}{\theta_H - \theta_L}. \qquad (A14)$$

Moreover,

$$\frac{\partial \hat{\theta}_2}{\partial \Delta} = \frac{u(N+2) - 0.5u(N+1) - 0.5u(N)}{2[u(N+2) - 0.75u(N+1) - 0.25u(N)]} < 1 \qquad (A15)$$

because

$$2[u(N+2)-0.75u(N+1)-0.25u(N)]-[u(N+2)-0.5u(N+1)-0.5u(N)]=$$

$$u(N+2)-u(N+1)>0. \qquad (A16)$$

Therefore

$$\frac{\partial \hat{p}_s}{\partial \Delta}<0. \qquad (A17)$$

Q.E.D.

Proposition 5. An increase in the employer's capacity to pay increases the expected settlement wages.

Proof. Under a parallel shift of the distribution of θ, the expected settlement wages,

$$E\hat{w}=\frac{\theta_H+\Delta-\hat{\theta}_2}{\theta_H-\theta_L}\hat{w}_2+\frac{\hat{\theta}_2-\hat{\theta}_1}{\theta_H-\theta_L}\hat{w}_1+\frac{\hat{\theta}_1-(\theta_L+\Delta)}{\theta_H-\theta_L}\bar{w} \qquad (A18)$$

where $\hat{w}_2$ and $\hat{w}_1$ are the new wage proposals of the union. Using equations (7), (11), and (12) it is straightforward to verify that

$$\frac{\partial \hat{w}_1}{\partial \Delta}>0 \qquad (A19)$$

and

$$\frac{\partial \hat{w}_2}{\partial \Delta}>0. \qquad (A20)$$

Proposition 4 verifies that the probability of a strike negatively depends on Δ. Therefore the complementary probablity of immediate settlement, $\frac{\theta_H+\Delta-\hat{\theta}_2}{\theta_H-\theta_L}$ positively depends on Δ. The probability of the one-period strike, $\frac{\hat{\theta}_2-\hat{\theta}_1}{\theta_H-\theta_L}$ also positively depends on Δ, because

$$\hat{\theta}_2-\hat{\theta}_1=\hat{\theta}_2-\left(\frac{\hat{\theta}_2}{2}+\frac{\bar{w}}{2[u(N+1)-u(N)]}\right)=\frac{\hat{\theta}_2}{2}-\frac{\bar{w}}{2[u(N+1)-u(N)]} \qquad (A21)$$

and $\frac{\partial \hat{\theta}_2}{\partial \Delta}>0$. Q.E.D.

Proposition 6. If the replacement share limit $\bar{\beta}$ is binding, there is a negative relationship between this limit and the union's settlement wage proposal.

Proof. Given that $\beta=\bar{\beta}$,

$$\tilde{w}_1=\frac{1}{N+1}[\tilde{\theta}_1(u(N+1)-u(N+\alpha\beta))+(N+\beta\gamma)\bar{w}]=$$

$$\frac{1}{N+1}[0.5\theta_H[u(N+1)-u(N+\alpha\beta)]+0.5(1-\beta\gamma)\bar{w}+(N+\beta\gamma)\bar{w}]. \qquad (A22)$$

Therefore,

$$\frac{\partial\tilde{w}_1}{\partial\beta}=\frac{1}{N+1}[-0.5\theta_H\alpha u'(N+\alpha\beta)+0.5\gamma\bar{w}] <$$

$$\frac{1}{N+1}[-0.5\theta_L\alpha u'(N+\alpha\beta)+0.5\gamma\bar{w}]<0. \qquad (A23)$$

The last inequality holds by condition (22). Q.E.D.

The impact of a change in the replacement share on the strike likelihood is generally ambiguous. Differentiating the probability of a strike equation, $p_s=\frac{\tilde{\theta}_1-\theta_L}{\theta_H-\theta_L}$, we get

$$\frac{\partial p_s}{\partial\beta}=\frac{1}{\theta_H-\theta_L}\frac{\partial\tilde{\theta}_1}{\partial\beta}. \qquad (A24)$$

Taking into account equation (21), we obtain

$$\frac{\partial p_s}{\partial\beta}=\frac{1}{\theta_H-\theta_L}\frac{-2\gamma\bar{w}[u(N+1)-u(N+\alpha\beta)]+2\alpha u'(N+\alpha\beta)(1-\beta\gamma)\bar{w}}{4[u(N+1)-u(N+\alpha\beta)]^2}. \qquad (A25)$$

The numerator of this equation may be greater or smaller than 0. However, for a particular class of utility functions this ambiguity can be resolved. Q.E.D.

Proposition 7. If $u'(L)$ is a non-decreasing function of L, there is a negative relationship between the replacement ratio and the probability of a strike.

Proof. If $u'(L)$ is a non-decreasing function of L, then applying the mean-value theorem,

$$u(N+1)-u(N+\alpha\beta)\geq u'(N+\alpha\beta)(1-\alpha\beta). \qquad (A26)$$

Therefore,

$$\frac{\partial p_s}{\partial\beta}\leq\frac{1}{\theta_H-\theta_L}2\bar{w}u'(N+\alpha\beta)\frac{-\gamma(1-\alpha\beta)+(1-\beta\gamma)\alpha}{4[u(N+1)-u(N+\alpha\beta)]^2}=$$

$$\frac{1}{\theta_H-\theta_L}2\bar{w}u'(N+\alpha\beta)\frac{\alpha-\gamma}{4[u(N+1)-u(N+\alpha\beta)]^2}<0. \qquad (A27)$$

The last inequality holds by the assumption that $\alpha<\gamma$. Q.E.D.

Proposition 8. There is a negative relationship between the replacement-wage parameter, γ, and the probability of a strike.

Proof: The probability of a strike,

$$p_s=\frac{\tilde{\theta}_1-\theta_L}{\theta_H-\theta_L}. \qquad (A28)$$

Differentiating the last equation with respect to γ yields:

$$\frac{\partial p_s}{\partial \gamma} = \frac{1}{\theta_H - \theta_L}\frac{\partial \tilde{\theta}_1}{\partial \gamma}. \tag{A29}$$

Differentiating equation (21) with respect to γ yields:

$$\frac{\partial \tilde{\theta}_1}{\partial \gamma} = -\frac{\beta \bar{w}}{2[u(N+1) - u(N+\alpha\beta)]} < 0. \tag{A30}$$

Hence,

$$\frac{\partial p_s}{\partial \gamma} = -\frac{1}{\theta_H - \theta_L}\frac{\beta \bar{w}}{2[u(N+1) - u(N+\alpha\beta)]} < 0. \tag{A31}$$

Q.E.D.

Proposition 9. There is a negative relationship between the replacement-wage parameter γ and the union's settlement wage proposal.

Proof: Differentiating equation (19) with respect to γ yields:

$$\frac{\partial \tilde{w}_1}{\partial \gamma} = \frac{\beta \bar{w}}{N+1} > 0. \tag{A32}$$

Q.E.D.

Proposition 10. If $u'(L)$ is a non-decreasing function of L, there is a positive relationship between the replacement ratio and social welfare.

Proof:

First, we show that for any school board, social welfare is higher if there is no strike.

From equation (27), we get:

$$W_{ns} - W_s = (\theta u(N+1) + \beta \bar{w}^r) - [\theta u(N+\alpha\beta) + \bar{w}] = \theta[u(N+1) - u(N+\alpha\beta)] - \bar{w} + \beta \bar{w}^r >$$

$$\theta_L(1-\alpha\beta)\bar{w} - \bar{w} + \beta\bar{w}^r = \beta(\bar{w}^r - \alpha\bar{w}) > 0. \tag{A33}$$

The first inequality holds by assumption (15), and the second one holds by assumption (16).

By Proposition 7, an increase in the replacement ratio β reduces the probability of a strike, i.e. lowers the threshold value of $\tilde{\theta}_1$. Let us denote the new value of the threshold (after the increase in β) as $\tilde{\theta}_1'$. Consider three ranges of values for θ:

1) $\theta_L \leq \theta < \tilde{\theta}_1'$; 2) $\tilde{\theta}_1' \leq \theta < \tilde{\theta}_1$; 3) $\tilde{\theta}_1 \leq \theta \leq \theta_H$.

The expected social welfare can be written as

$$EW = \int_{\theta_L}^{\tilde{\theta}_1'} W(\theta)d\theta + \int_{\tilde{\theta}_1'}^{\tilde{\theta}_1} W(\theta)d\theta + \int_{\tilde{\theta}_1}^{\theta_H} W(\theta)d\theta. \qquad (A34)$$

School boards, whose type is in the range 1, endure a strike before and after the change in β. Hence,

$$\int_{\theta_L}^{\tilde{\theta}_1'} W(\theta)d\theta = \int_{\theta_L}^{\tilde{\theta}_1'} [\theta u(N + \alpha\beta) + \bar{w}]d\theta. \qquad (A35)$$

The right-hand side of the equation positively depends on β, because $u'(.) > 0$.

School boards of intermediate type (range 2) endure strike only before the change in β. Given that social welfare is higher if there is no strike, $\int_{\tilde{\theta}_1'}^{\tilde{\theta}_1} W(\theta)d\theta$ (the second term of the equation) positively depends on β.

School boards of the high type (range 3) do not endure a strike either before, or after the change in β. Hence,

$$\int_{\tilde{\theta}_1}^{\theta_H} W(\theta)d\theta = \int_{\tilde{\theta}_1}^{\theta_H} [\theta u(N + 1) + \beta\bar{w}^r]d\theta. \qquad (A36)$$

The right-hand side of the equation positively depends on β.

Therefore, each term of the right-hand side of equation is higher when the replacement ratio, β, is higher. Therefore, the expected value of the social welfare is higher as well. Q.E.D.

Proposition 11. There is a positive relationship between the replacement-wage parameter, γ, and social welfare.

Proof: By Proposition 8, an increase in the replacement-wage parameter γ reduces the probability of a strike, i.e. lowers the threshold value of $\tilde{\theta}_1$. Let us denote the new value of the threshold (after the increase in γ) as $\tilde{\theta}_1'$. Consider three ranges of values for θ:

1) $\theta_L \leq \theta < \tilde{\theta}_1'$; 2) $\tilde{\theta}_1' \leq \theta < \tilde{\theta}_1$; 3) $\tilde{\theta}_1 \leq \theta \leq \theta_H$.

The social welfare across school board types can be written as:

$$EW = \int_{\theta_L}^{\tilde{\theta}_1'} E(W(\theta))d\theta + \int_{\tilde{\theta}_1'}^{\tilde{\theta}_1} E(W(\theta))d\theta + \int_{\tilde{\theta}_1}^{\theta_H} E(W(\theta))d\theta. \qquad (A37)$$

School boards, whose type is in the range 1, endure a strike before and after the change in γ. Hence,

$$\int_{\theta_L}^{\tilde{\theta}'_1} E(W(\theta))d\theta = \int_{\theta_L}^{\tilde{\theta}'_1} [\theta u(N + \alpha\beta) + \bar{w}]d\theta. \qquad (A38)$$

The right-hand side of the last equation is independent of γ.

School boards of intermediate type (range 2) endure strike only before the change in γ. Given that social welfare is higher when there is no strike for each type of the school board, $\int_{\tilde{\theta}'_1}^{\tilde{\theta}_1} E(W(\theta))d\theta$ (the second term of the equation) positively depends on γ.

School boards of the high type (range 3) do not endure a strike either before, or after the change in β. Hence,

$$\int_{\tilde{\theta}_1}^{\theta_H} E(W(\theta))d\theta = \int_{\tilde{\theta}_1}^{\theta_H} [\theta u(N + 1) + \beta\bar{w}^r]d\theta. \qquad (A39)$$

The right-hand side of the last equation is independent of γ.

Thus, the first and the third terms of the right-hand side of the equation do not change when the replacement ratio, γ, is higher, while the second term increases. Therefore, the social welfare across board types increases as well. Q.E.D.

Appendix B. Models under Low Uncertainty

Solutions of the Benchmark Model and Extended Model under low uncertainty follow.

Benchmark Model under Low Uncertainty

Assume that condition (13) does not hold. In other words, at the beginning of the second period the union cannot benefit from discriminating among different school board types. Instead it makes the offer w_1 just acceptable to the lower type, θ_L. This offer is accepted by all school boards in the range $[\theta_L, \theta_2]$

$$\theta_L u(N+1) - (N+1)\hat{w}_1 = \theta_L u(N) - N\bar{w}. \qquad (B1)$$

Solving this equation for $\hat{w}_1$ yields

$$\hat{w}_1 = \frac{1}{N+1}[\theta_L(u(N+1) - u(N)] + N\bar{w}]. \qquad (B2)$$

At the beginning of the first period, the problem that the union faces is very similar to the problem it faces at the beginning of the second period in the game with high uncertainty. The union maximizes the expected payoff

$$\hat{\Pi}_2 = \frac{\theta_H - \hat{\theta}_2}{\theta_H - \theta_L}(N+2)\hat{w}_2 + \frac{\hat{\theta}_2 - \theta_L}{\theta_H - \theta_L}[(N+1)\hat{w}_1 + \bar{w}], \qquad (B3)$$

where $\hat{\theta}_2$ is the type of the school board which is indifferent between accepting and rejecting the offer. The following equation describes the indifference condition

$$\hat{\theta}_2 u(N+2) - (N+2)\hat{w}_2 = \hat{\theta}_2 u(N+1) - (N+1)\hat{w}_1. \qquad (B4)$$

The school board maximizes the objective function with respect to $\hat{w}_2$ subject to the constraint. Maximization yields

$$\hat{w}_2 = \frac{1}{N+2}\left[0.5\theta_H[u(N+2) - u(N+1)] + 0.5\bar{w} + (N+1)\hat{w}_1\right]. \qquad (B5)$$

Replacing $\hat{w}_2$ in the indifference condition, we get

$$\hat{\theta}_2 = \frac{\theta_H}{2} + \frac{\bar{w}}{2[u(N+2) - u(N+1)]}. \qquad (B6)$$

As long as

$$\hat{\theta}_2 = \frac{\theta_H}{2} + \frac{\bar{w}}{2[u(N+2) - u(N+1)]} > \theta_L, \tag{B7}$$

the Perfect Bayesian equilibrium of the game is still a separating one, i.e. strikes are possible, if the realization of θ falls below $\hat{\theta}_2$. However, if this condition is violated, the equilibrium of the game is a pooling one, i.e. it is not beneficial for the union to discriminate among different school board types at all. It makes an offer $\hat{w}_2$ just acceptable to the school board of the type θ_L, and hence accepted by all types.

$$\theta_L u(N+2) - (N+2)\hat{w}_2 = \theta_L u(N+1) - (N+1)\bar{w}. \tag{B8}$$

Solving the equation for $\hat{w}_2$ yields

$$\hat{w}_2 = \frac{1}{N+2}[\theta_L[u(N+2) - u(N+1)] + (N+1)\bar{w}]. \tag{B9}$$

Extended Model under Low Uncertainty

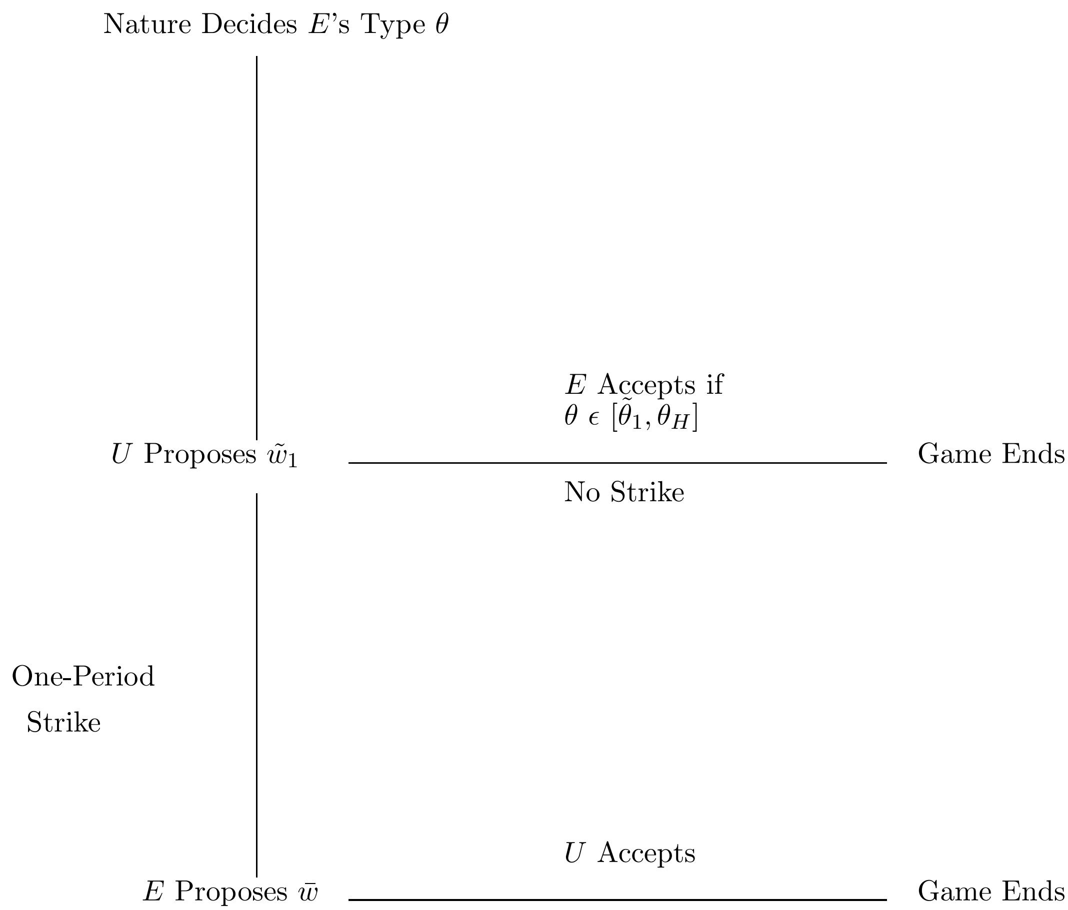
If condition (19) is violated, it is not beneficial for the union to discriminate among different school board types. It makes an offer $\hat{\tilde{w}}_1$ just acceptable to the school board of type θ_L.

$$\theta_L u(N+1) - (N+1)\hat{\tilde{w}}_1 = \theta_L u(N+\alpha\beta) - \beta\gamma\bar{w} - N\bar{w}. \tag{B10}$$

Solving the equation for $\hat{\tilde{w}}_1$ yields

$$\hat{\tilde{w}}_1 = \frac{1}{N+1}[\theta_L[u(N+1) - u(N+\alpha\beta)] + (N+\beta\gamma)\bar{w}]. \tag{B11}$$

Figure 1: Bargaining Process with Replacement Teachers



Note: E = School Board, U = Regular Teachers' Union.

References

Card, D. (1990), "Strikes and Wages: A Test of an Asymmetric Information Model," *The Quarterly Journal of Economics* 105, 625–659.

Cramton, P., Gunderson, M. and Tracy, J. (1999), "The Effect of Collective Bargaining Legislation on Strikes and Wages," *Review of Economics and Statistics* 81, 475–487.

Cramton, P. and Tracy, J. (1998), " The Use of Substitute Workers in Union Contract Negotiations: The U.S. Experience, 1980-1989," *Journal of Labor Economics* 16, 667–701.

Currie, J. (1991), "Employment Determination in a Unionized Public- Sector Labor Market: The Case of Ontario's School Teachers," *Journal of Labor Economics* 9, 45–66.

Dilts, D. (1986), "The Negotiation of Teacher Economic Packages: An Analysis of Kansas' Settlements for 1983 and 1984," *Journal of Collective Negotiations* 15, 273–280.

Galbally, E. (2002), "Red Wing Teachers Are on Strike," Minnesota Public Radio, October 22.

Gertner, R. and Miller, G. (1995), "Settlement Escrows," *Journal of Legal Studies* 24, 87–122.

Gold, R. (2001), " Your Career Matters: Schools Grapple With Shortage Of Substitutes," *Wall Street Journal* (Eastern Edition), January 23, B.1.

Gul, F., Sonnenschein, H., and Wilson, R. (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory* 39, 155–190.

Hart, O. (1986), "Bargaining and Delay, " manuscript. Massachusetts Institute of Technology.

Ichniowshki, C. (1982), "Arbitration and Police Bargaining: Prescription for the Blue Flue," *Industrial Relations* 21, 149–166.

Pennsylvania School Board Association (1997), " Collective Bargaining Success of Act 88," PSBA White Paper on Relief From Mandates, March.

Pittsburgh Post Gazette (1990), "South Allegheny Teachers Out," November 8.

The Christian Science Monitor (2002), "Substitute!," November 12.

The School Administrator Web Edition (2001), "On Deck: The Unionization of Substitutes," January.

Toppo. G. (2001), "Back to School: With Teacher Shortage Worsening, Schools Rely on Substitutes," Associated Press, August 28.

Tracy, J. (1987), "An Empirical Test of an Asymmetric Information Model of Strikes," *Journal of Labor Economics* 5, 149–173.

Tracy, J. and Gyourko, J. (1991), "Public Sector Bargaining and the Local Budgetary Process," *Research in Labor Economics* 12, 117–136.

Zwerling, H. and Thomason, T. (1995), "Collective Bargaining and the Determinants of Teachers' Salaries," *Journal of Labor Research* 16, 467–484.