Name:_____

Intermediate Macroeconomic Theory II, Fall 2010
Instructor: Dmytro Hryshko
Problem Set 1 (40 points). Due October 15

1. (**9 points**) Consider the Permanent Income Hypothesis we studied in class. Preferences are quadratic, $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$; planning horizon is infinite; $\beta(1+r) = 1$; income stream is known as of time 0. Consider two individuals, X and Y, with the same present discounted value of incomes. Individual X's income starts at 100 at time 0 and grows at the net rate 2% each year, $g_X = 0.02$, while individual Y's income starts at 300 at time 0 and falls at the net rate –2% each year, $g_Y = -0.02$. The real interest rate, r, equals 4%.

   (a) (**2 points**) Show that my claim that individual X's and Y's present discounted values of income are indeed the same, and find the values they are equal to.

   (b) (**2 points**) Find the optimal consumption for individual X and Y in each period, assuming that both are not borrowing constrained.

(c) (**1 point**) Assume that individual X cannot borrow but can save at the interest r equal 4%. What is X's optimal level of consumption in this case?

(d) (**2 points**) Assume that individual Y cannot borrow but can save at the interest r equal 4%. What is Y's optimal level of consumption now?

(e) (**2 points**) (For this question, you don't need to show any algebra.) Assume now that individual X doesn't know his future incomes at time t. Would s/he set consumption to a constant level for times t, $t+1$, etc.? If you observe a large change in individual X's consumption between periods t and $t+1$, what could you infer about individual's circumstances assuming that the only thing that happens to this individual is changes in productivity?

2. (**10 points**) Assume the "Solow" economy is currently in the steady state. The savings rate $s = 20\%$; production function is $Y = K^{4/10}L^{6/10}$; population growth rate, n and depreciation rate, δ are some positive constants. (You don't need to know what they are to answer the questions, but you may pick any numbers if it helps.) The government decides to tax both wage and capital income at the proportional rate $0 < \tau < 1$. Thus, consumers receive real income in the amount equal to $(1 - \tau)Y$. Assume that the government invests the full amount of the tax proceedings. Thus, government savings are τY.

   (a) (**5 points**) Would you observe any changes in the economy resulting from the tax policy and, if yes, <u>what</u> changes (steady-state output per worker? steady-state capital per worker?)? Show a graph.

   (b) (**5 points**) If the governments wants to set τ to move the economy towards the golden-rule steady-state, what value of τ would it choose?

3. (**21 points**) Let the economy's production function be $Y = 2K^{1/2}(EL)^{1/2}$.
Households save 40% of their income;
population growth, n, is equal to 1%;
the depreciation rate, δ, is equal to 2%;
the growth rate in the efficiency of labor, g, is 1%.

   (a) (**2 points**) Show that the aggregate production function is constant returns to scale in K and L (**1 point**), and express the production function in *per-effective-worker* terms (**1 point**).

   (b) (**1 point**) Is production function increasing/constant/decreasing returns to scale in 3 factors of production, K, E, and L? *Show* how you arrived at the conclusion.

   (c) (**3 points**) Calculate the steady state level of
   capital *per effective worker* (**1 point**)
   output per *effective worker* (**1 point**)
   and consumption *per effective worker* (**1 point**).

(d) (**1 point**) If you were a social planner who maximizes consumption per worker in the economy, what savings rate would you choose? (You *need not* show your calculations here if you see the answer.)

(e) (**2 points**) Find the golden rule level of capital per effective worker (**1 point**) and the corresponding output per effective worker in the golden rule steady state (**1 point**).

(f) (**3 points**) Calculate the real interest rate under the economy's current savings rate (40%), when the economy is in the steady state. (**1 point**)
Calculate the real interest rate when the economy is in the golden rule steady state. (**1 point**)
If the interest rates differ, briefly argue why one is higher than another. (**1 point**)

(g) (**1 point**) Calculate the real wage on a balanced growth path, when the economy's savings rate is 40%. (*Hint*: the real wage at time t will be a function of E at time t.)

(h) (**1 point**) What is the growth rate of total output in the steady state (on a balanced growth path, to be precise)?

(i) (**1 point**) What is the growth rate of the real wage in the economy? What is the growth rate of the real interest rate?

(j) (**1 point**) What is the share of capital and labor costs in total income?

(k) (**2 points**) Assume the economy is on a balanced growth path. Let the production function be $Y = BK^{1/2}L^{1/2}$, where $B = 2E^{1/2}$, and B is the total factor productivity. What is the contribution of the total factor productivity towards the growth in total output? That is, calculate $\frac{\Delta B/B}{\Delta Y/Y}$.

(l) (**2 points**) Assume economy is on a balanced growth path, where s, n, g and δ are as before. Assume that at some time T savings rate is increased permanently from 40% to 50%. Calculate the growth rate of real wages at time T. (**1 point**) What is the growth rate of real wages when the economy is on its new balanced growth path? (**1 point**)

(m) (**1 point**) Is the following statement true or not? For the just described economy, consumption per worker, as well as capital per worker and output per worker, will be higher in the steady state with the savings rate equal to 40% rather than 50%.